We can define the vertex function $\Gamma_\nu^{i\alpha}$ by

$$\sum_\beta {}_0\langle T\psi_\alpha(x)g_{i\beta}J_\mu{}^\beta(z)\bar{\psi}_\alpha(y)\rangle_0$$
$$=-\int\int d^4ud^4u' S_\alpha(x-u)$$
$$g_{i\alpha}\Gamma_\mu{}^{(i\alpha)}(u-z;z-u')S_\alpha(u'-y). \quad (5.5)$$

Taking the divergence and then the Fourier transform of this equation, we obtain the Ward-Takahashi identity[11,12] for the field A^i interacting with the current J^α:

$$S_\alpha{}^{-1}(p)-S_\alpha{}^{-1}(q)=(p-q)_\nu\Gamma_\nu{}^{i\alpha}(p,q), \quad (5.6)$$

and thus,

$$\partial S_\alpha{}^{-1}(p)/\partial p_\nu=\Gamma_\nu{}^{i\alpha}(p,p). \quad (5.7)$$

The renormalized functions $\tilde{\Gamma}_\nu{}^{i\alpha}$, $\tilde{S}_\alpha$ are given by

$$\tilde{S}_\alpha=Z_{2\alpha}{}^{-1}S_\alpha, \quad (5.8)$$

$$g_{i\alpha}\Gamma_\nu{}^{(i\alpha)}=Z_{2\alpha}{}^{-1}R^{-1}{}_{ki}\tilde{g}_{k\alpha}\tilde{\Gamma}_\nu{}^{k\alpha}. \quad (5.9)$$

Therefore,

$$Z_1{}^{-1}{}_{i\alpha}\tilde{\Gamma}_\nu{}^{(i\alpha)}(p,p)=Z_{2\alpha}{}^{-1}\partial\tilde{S}_\alpha{}^{-1}/\partial p_\nu, \quad (5.10)$$

but

$$\tilde{\Gamma}_\nu{}^{i\alpha}(p^2=q^2=M_\alpha{}^2,\ (p-q)^2=M_i{}^2)$$
$$=(1+L_{i\alpha})\gamma_\nu=(2-Z_{1i\alpha})\gamma_\nu \quad (5.11)$$

and

$$\partial\tilde{S}_\alpha{}^{-1}/\partial p_\nu(p^2=M_\alpha{}^2)=(1-B_{i\alpha})\gamma_\nu=(2-Z_{2\alpha})\gamma_\nu; \quad (5.12)$$

therefore,

$$\text{if } M_i=0,\quad Z_{1i\alpha}=Z_{2\alpha}. \quad (5.13)$$

Then, from Eqs. (3.5) and (5.2), we deduce that

$$\tilde{g}_{1\alpha}=R_{11}g_{1\alpha},\quad \alpha=1,\cdots,N. \quad (5.14)$$

This shows that the electric charges of all particles are changed by the same factor when renormalized, as we asserted.

CONCLUSION

We have shown that when mixing occurs, the renormalized fields must be taken as linear combinations of the bare fields. In order to calculate matrix elements, observed masses and coupling constants are used, but mixing parameters are not needed. We use a propagator whose pole terms are diagonal, and subtract self-energy parts at each relevant mass in order to calculate it. To lowest order this method gives the same results as the prescription of Feldman and Matthews,[1] and also calculates higher order corrections correctly.

Finally we have shown that photon–vector-meson mixing still allows a zero bare mass for the photon, and that in such circumstances the electric charges of different particles are again renormalized by the same factor.

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[11] Y. Takahashi, Nuovo Cimento **6**, 371 (1957).
[12] J. Bernstein, M. Gell-Mann, and L. Michel, Nuovo Cimento **16**, 560 (1960).

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General Relativistic Instability*

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The point of instability of a general relativistic fluid sphere is determined using the criterion that the point of instability occurs when the binding energy is at a maximum. The result is equivalent to the result obtained by the small-perturbation method when the radius is sufficiently large.

A RECENT paper[1,2] has shown, using the method of small perturbations, that gaseous masses may exhibit a radial instability in the framework of general relativity. When discussing instability in Newtonian physics, one sometimes uses an energy method to determine the point of instability, and it can be shown that in certain cases the methods are equivalent. For the case of a general relativistic fluid sphere with constant energy density and heat capacity, one can show that the two methods give exactly equivalent results at the limit where the radius is much greater than the Schwarzschild radius. The instability is assumed to occur when the binding energy of the system is at a maximum.

The expression for the binding energy of a static,

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[1] S. Chandrasekhar, Phys. Rev. Letters **12**, 114 (1964).
[2] S. Chandrasekhar, Phys. Rev. Letters **12**, 437 (1964).

spherically symmetric fluid sphere is[3,4]

$$\mathrm{B.E.}=4\pi\int_0^R \rho_m c^2 e^{\lambda/2} r^2 dr - 4\pi\int_0^R (T_0{}^0 - T_1{}^1 - T_2{}^2 - T_3{}^3) e^{(\lambda+\nu)/2} r^2 dr. \quad (1)$$

The first term expresses the potential energy of the system and the last term the total energy of a system with a metric

$$ds^2 = e^\nu dx_0{}^2 - e^\lambda dr^2 - r^2(\sin^2\theta d\phi^2 + d\theta^2), \quad (2)$$

where

$$\begin{aligned} &e^\nu = \tfrac{1}{4}(3y_1 - y)^2, \quad e^{-\lambda} = y^2, \\ &T_0{}^0 = \epsilon_0, \quad T_1{}^1 = T_2{}^2 = T_3{}^3 = -p, \\ &p/\epsilon_0 = (y-y_1)/(3y_1 - y), \quad y_1{}^2 = 1 - qR^2, \\ &y^2 = 1 - qr^2, \quad q = \tfrac{1}{3}8\pi k\epsilon_0 c^{-4}, \end{aligned} \quad (3)$$

and

$$M = 4\pi c^{-2}\int_0^R \epsilon_0 r^2 dr.$$

The quantity ϵ_0 is the energy density, ρ_m the mass density, p the pressure, R the radius of the system, and k the gravitational constant.

A gas sphere containing radiation and monatomic gas only would have an energy density

$$\epsilon_0 = \rho_m c^2 + 3p(1-\beta) + \tfrac{3}{2}\beta p, \quad (4)$$

where β is ratio of particle pressure to total pressure. The use of Eqs. (2), (3), and (4) enables one to express the binding energy as

$$\mathrm{B.E.} = 4\pi\int_0^R \epsilon_0[1 - y - 3(1-\tfrac{1}{2}\beta)(y-y_1) \times (3y_1 - y)^{-1}] y^{-1} r^2 dr. \quad (5)$$

[3] I. Iben, Astrophys. J. **138**, 4, 1090 (1963).

[4] L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Inc., New York, 1962), 2nd ed., Sec. 100.

TABLE I. The critical radius R_c in units of the Schwarzschild radius for various values of β, for which the equivalent ratio of specific heats is given.

β	Γ_1	R_c/R_0
0.290	1.384	9.0
0.306	1.387	8.549
0.645	1.461	4.0
0.828	1.523	3.0
0.923	1.579	2.7
0.954	1.609	2.6
1.000	1.667	2.465

Equation (5) may be integrated for the case β= constant. The requirement $\partial\,\mathrm{B.E.}/\partial R = 0$, subject to the condition M= constant, then becomes

$$x^{-1/2}\sin^{-1}x^{1/2} - (1-\tfrac{1}{3}x)(1-x)^{-1/2} + (1-\tfrac{1}{2}\beta) \times \Big\{ (2/x)^{1/2}(1-x)^{-1/2}(1-9x/8)^{-1/2}(24-34x+9x^2) \times \Big[\frac{\pi}{2} - \sin^{-1}(1-\tfrac{3}{2}x)(1-x)^{-1/2}\Big] - 15(1-\tfrac{1}{3}x)(1-x)^{-1/2} - x^{-1/2}(33-12x)\sin^{-1}x^{1/2}\Big\} = 0, \quad (6)$$

where $x = R_0/R$ and $R_0 = 2kMc^{-2}$ (the Schwarzschild radius).

Equation (5) reduces to

$$\mathrm{B.E.} = (3kM^2/10R)(\beta - 19R_0/14R) \quad (7)$$

for the case $(R_0/R)\ll 1$. The point of instability occurs at $R_c = 19R_0/7\beta$. An object which is nearly all radiation has[5] $\beta \cong 6(\Gamma_1 - \tfrac{4}{3})$, and the point of instability occurs at $R_c = 19R_0/42(\Gamma_1 - \tfrac{4}{3})$, which is identical to the result obtained by the method of small perturbations.[2] Values of the critical radius for various values of β using Eq. (6) are given in Table I.

[5] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure* (University of Chicago Press, Chicago, 1939), p. 57.